

# Critical Spectral Statistics at the Metal-Insulator Transition in Interacting Fermionic Systems

Philippe Jacquod

Department of Applied Physics, P. O. Box 208284, Yale University, New Haven, CT 06520-8284  
(june 29, 1998)

The spectral properties of a disordered system with few interacting three-dimensional spinless fermions are investigated. We show the existence of a critical spacings distribution which is invariant upon increase of the system size, but strongly depends on the number of particles. At the critical point, we report a substantial decrease of the degree of level repulsion as the number of particles increases indicating a decrease of nearest level correlations associated with the sparsity of the Hamiltonian matrix.

PACS numbers: 05.45.+b, 72.15.Rn, 71.30.+h

The presence and strength of a disorder potential strongly modify the influence of electronic interactions on localization. While in clean systems repulsive interactions always localize, their effect can be reversed by the presence of a strong disorder. There, interaction-induced hoppings between localized states may help electrons to overcome the random potential thus reducing localization [1–3]. In this paper we will show that repulsive interactions can indeed trigger a metal-insulator transition in a few-particle strongly disordered system, thereby turning a one-body insulator into a many-body metal.

The statistical properties of spectra of disordered one-particle systems are known to be closely related to the localization properties of the corresponding eigenstates [4–6]. In the localized phase, states which are close in energy have an exponentially small overlap. Consequently the levels  $E_i$  are uncorrelated and the corresponding normalized spacings  $s_i \equiv (E_{i+1} - E_i)/\langle E_{i+1} - E_i \rangle$  are distributed according to the Poisson distribution  $P_P(s) = \exp(-s)$ . On the other hand, the large overlap of delocalized eigenstates of neighboring energy induces correlations in the spectrum and leads to level repulsion, so that in the time-reversal symmetric case, the distribution of energy spacings corresponds to the Wigner-Dyson distribution  $P_{WD}(s) = \frac{\pi}{2}s \exp(-\pi s^2/4)$  [4]. While these two limiting behaviors set in progressively as the linear system size  $L$  increases, a third, size-independent distribution appears at the metal-insulator transition due to the divergence of the correlation length [5]. This behavior suggested a new powerful method to locate the metal-insulator transition in one-particle models, relying only on the knowledge of the spectrum to deduce the localization properties of the eigenfunctions. It is one of the purposes of the present letter to generalize this method to many-body interacting disordered systems.

Spectral properties of many-body systems have already attracted some interest. Early works showed that, for clean systems, only non-generic models where special group symmetries ensure the integrability, have a Poissonian level spacings distribution [7]. In the presence

of disorder, it has been suggested that the interaction induces correlations between many-body wave functions and in the corresponding spectrum, thus resulting in level repulsion. This phenomenon has been related to a delocalization in real space in the Coulomb glass [3] and in a system of two interacting particles in a disordered potential [8], and to the recently intensively investigated *delocalization in Hilbert/Fock space* [9–11]. The latter concept refers to an interaction-induced spreading of the many-body states over the basis of Slater determinants built out of the one-particle states. For sufficiently large interactions, this spreading covers many basis states whose number represents a localization length in Hilbert/Fock space much in the usual sense. The local spectral density typically acquires a Breit-Wigner shape  $\rho_{BW}(E) = \Gamma/(2\pi(E^2 + \Gamma^2/4))$  and the spreading of many-body states may be characterized by the width  $\Gamma$  [8,12]. The latter may be estimated from the Golden Rule which gives  $\Gamma \sim Q^2/\Delta_c$  where  $Q^2$  is the mean square interaction matrix element and  $\Delta_c$  the spacings between states directly coupled by the two-body interaction. Thus the interaction starts to mix directly coupled states as the critical threshold  $Q_c \sim \Delta_c$  is reached and simultaneously, spectral correlations result in level repulsion and the emergence of quantum chaos [10]. Quite surprisingly, it was shown in [11] that the number of basis states occupied by a single interacting state is given by  $\xi \sim \Gamma/\Delta_n$  where  $\Delta_n$  is the  $n$ -particle mean spacing. As quantum chaos starts to set in one has  $\xi_c \sim \Delta_c/\Delta_n \gg 1$  so that contrarily to the common belief, level repulsion appears *after* many noninteracting states have been mixed by the interaction. Since the models studied in [10,11] contained no information on the real space, no conclusion could be drawn as to a transition in real space associated to the emergence of quantum chaos or the mixing of eigenstates, a question which is of particular interest.

In this letter, we present a systematic investigation of the spectral properties of a system of few interacting quasiparticles above a frozen (i.e. noninteracting) Fermi sea. The neglect of interaction processes involv-

ing one-particle states located in the frozen Fermi sea is advantageous in two respects : (i) it allows to reduce significantly the Hilbert space volume and consequently to study large systems of linear sizes up to  $L = 10$  in three dimensions (Presumably, it is the impossibility of studying systems with large enough sizes that hindered investigations similar to [5] in interacting systems so far.) and (ii) it projects out one-body tail states which are known to be strongly localized and hence much less sensitive to the interaction-induced delocalizing effect. For the two-quasiparticle case, this model was originally proposed in [2] and further studied in [13].

More specifically, we consider spinless fermions in a disordered Anderson-like model with a repulsive nearest neighbor interaction of strength  $U$  whose Hamiltonian is

$$H = P_b \left[ \sum_i W_i a_i^\dagger a_i + \sum_{\langle i;j \rangle} (U a_i^\dagger a_j^\dagger a_j a_i - t a_i^\dagger a_j) \right] P_b \quad (1)$$

Here,  $P_b$  projects out Slater determinants containing one-particle states of energy lower than the threshold energy  $E_b$ .  $\langle i;j \rangle$  restricts the sum to nearest-neighbor sites on a three-dimensional cubic lattice and  $W_i$  is the on-site disorder randomly distributed between  $-W/2$  and  $W/2$ . In our numerical investigations we fixed  $E_b \approx -3.3t$  corresponding to a filling factor of  $\nu = 1/3$  in the disorder regime we will consider  $W = 18$ . We point out that this choice is arbitrary and checked numerically that the physical picture presented here does not depend on this particular choice of  $E_b$ . For a system of linear size  $L$  and  $n$  quasiparticles, the ground-state energy is approximately  $\epsilon_g \approx nE_b + n(n+1)\Delta_1/2 + 3Un(n-1)\Delta_1/B_1$ .  $\Delta_1 \approx B_1/L^d$  is the one-particle average spacings and  $B_1 \approx 2dt + W$  is the one-particle bandwidth. We also define the many-body excitation energy as  $\delta\epsilon \equiv \epsilon - \epsilon_g$ . As the introduction of the frozen Fermi sea may appear somehow artificial, we stress that for low many-body energy excitations  $\delta\epsilon$  and moderate interaction  $U \ll B_1$ , only particles in an energy layer of size  $T \sim \sqrt{\delta\epsilon\Delta_1}$  around the Fermi level effectively interact and their number is given by  $n_{eff} \sim T/\Delta_1 \sim \sqrt{\delta\epsilon/\Delta_1}$  so that for  $n_{eff} < n$  our model is fully justified.

The numerical investigations were carried out much in the same way as in reference [5,10]. We computed the distribution  $P(s)$  for the many-body level spacings  $s_i \equiv (\epsilon_{i+1} - \epsilon_i)/\Delta_n$  for  $n = 2 - 6$  quasiparticles, at fixed excitation energies and disorder, for different interaction strengths and linear system size. As the interaction strength increases, more and more states are coupled and we expect quantum chaos to set in, i.e. the level spacings distribution undergoes a crossover from Poisson to Wigner-Dyson distribution. To quantitatively study this crossover we computed the value of the parameter  $\eta = \int_0^{s_0} [P(s) - P_{WD}(s)] ds / \int_0^{s_0} [P_P(s) - P_{WD}(s)] ds$ . Here,  $s_0 \approx 0.473$  is the smallest root of  $P_P(s) = P_{WD}(s)$  and  $\eta$  varies between 1 ( $P(s) = P_P(s)$ ) and 0 ( $P(s) =$

$P_{WD}(s)$ ). All investigations were carried out at fixed disorder  $W = 18$ , slightly above the critical disorder  $W_c \approx 16.5$  of the Anderson model with boxed distributed disorder [5,14]. At this disorder, one-particle states still have a large enough localization length  $l_1$  so that the short-range interaction couples a large number of states. We averaged over up to 10000 disorder realisations, so as to get at least  $N = 50000$  levels in an energy interval  $[\delta\epsilon - \Delta_\epsilon/2; \delta\epsilon + \Delta_\epsilon/2]$ ,  $\Delta_\epsilon < \Delta_1$ .

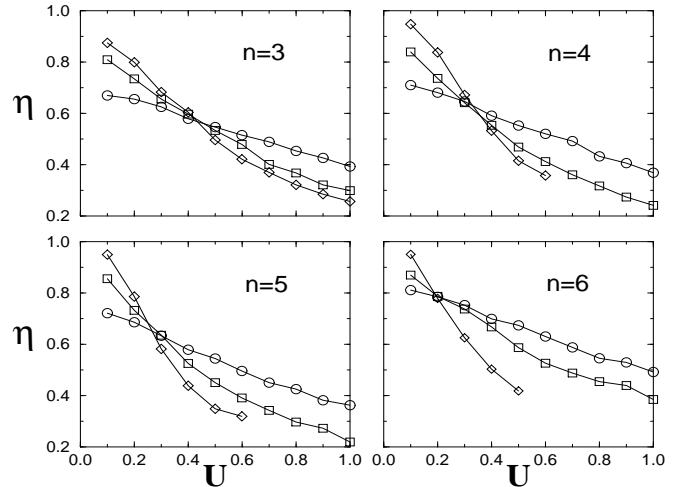


FIG. 1. Evolution of the crossover parameter  $\eta$  for  $n = 3$ ,  $\delta\epsilon = 0.35$ ,  $n = 4$ ,  $\delta\epsilon = 0.5$ ,  $n = 5$ ,  $\delta\epsilon = 0.65$  and  $n = 6$ ,  $\delta\epsilon = 0.8$  and disorder  $W = 18$ . (From left to right and top to bottom). The linear system sizes are  $L = 6$  (circles),  $L = 8$  (squares) and  $L = 10$  (diamonds). For each  $n$ , excitation energies satisfy  $n_{eff} \approx 2\sqrt{\delta\epsilon/\Delta_1} \approx n$  for  $L = 6$ .

Fig.1 shows the dependence of the crossover parameter  $\eta$  as a function of  $U$  for different number of particles. Clearly, the situation is similar to the one encountered in [5] for the Anderson transition : the  $\eta$ -curves belonging to different system sizes intersect at a critical  $U_c$  where  $\eta_c$  is size-independent. This indicates the existence of a scaling due to the divergence of a correlation length associated to the many-body eigenstates: there is a continuous transition from an insulating phase at  $U < U_c$  where  $\eta$  increases with the system size to a metallic one at  $U > U_c$  where the opposite behavior is observed. We stress that, as already observed in different strongly disordered systems [1,2] and unlike in clean ones, the presence of a repulsive interaction *favours* delocalization.

The existence of this transition dictates the value of  $\eta_c$  which consequently depends on the number of particles. Thus the critical distribution, i.e. the degree of level repulsion also depends on the number of particles, as shown in Fig.2.  $\eta_c(n)$  is a monotonously increasing function of  $n$  and it is tempting to say that as  $n \rightarrow \infty$ , the critical distribution tends to  $P_P(s)$ , i.e. *there is no level repulsion at the metal-insulator transition in a system with an infinite number of interacting quasiparticles*.

The increase of  $\eta_c(n)$  with  $n$  and the corresponding

decrease in the degree of correlations between nearest levels may indicate that this metal-insulator transition proceeds much in the same way as the transition to chaos reported in [10], i.e. that many-body delocalization results from the mixing of directly coupled states and that transport in the metallic phase proceeds by interaction-induced jumps between these states. In this case, the critical interaction satisfies the condition  $Q_c \sim \Delta_c$ . Accordingly, only states separated by an energy interval  $\Delta_c \gg \Delta_n$  are correlated and the resulting level repulsion can be distributed over  $N_n \equiv \Delta_c/\Delta_n$  successive level spacings. Hence if  $N_n \rightarrow \infty$ , as  $n \rightarrow \infty$ , the level repulsion vanishes at the transition.

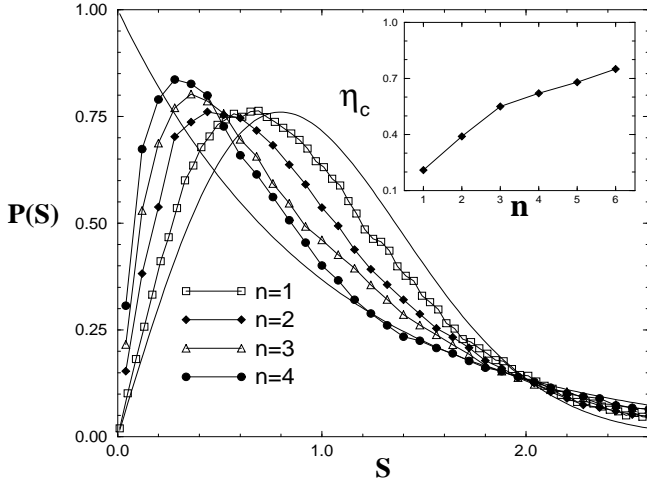


FIG. 2. Critical distributions for  $n = 1$  (squares),  $n = 2$  (full diamonds),  $n = 3$  (triangles) and  $n = 4$  (full circles). Also shown are the Wigner-Dyson and the Poisson distribution (full lines). Inset : Evolution of the critical crossover parameter  $\eta_c$  as a function of the number of particles.

We can estimate a lower boundary for  $N_n$  following a similar argument as in [10]. Assuming that at the transition, the system is thermalized, we consider  $n \rightarrow \infty$  particles at a temperature  $1 \ll T/\Delta_1 \ll n$ . The number of effectively interacting particles is then  $n_{eff} \sim T/\Delta_1$  so that they may be distributed over  $m \approx 2n_{eff}$  different one-body levels. The two-body interaction connects each state to roughly  $K \approx n_{eff}(n_{eff}-1)(m-n_{eff})(m-n_{eff}-1)/4 \sim n_{eff}^3(T/\Delta_1)$  states. All these possible transitions occur in an energy interval  $B_2 \approx (2m-4)\Delta_1 \approx 4n_{eff}\Delta_1$  so that in agreement with [10]

$$\Delta_c \sim \frac{B_2}{K} \sim \frac{\Delta_1^2}{n_{eff}^2 T} \quad (2)$$

Next we have  $\Delta_n \approx 2\Delta_1^n(n-1)!/\delta\epsilon^{n-1}$  (as before  $\delta\epsilon \approx T^2/\Delta_1$ ) and consequently  $N_n$  is exponentially large

$$\ln(N_n) \approx n_{eff} \ln(n_{eff}) \quad (3)$$

This latter result means in particular that at the transition, level correlations are distributed over a fast in-

creasing number of many-body levels, and thus explains how the level repulsion is reduced by an increasing number of interacting particles. Equation (3) follows from the increasing sparsity of the Hamiltonian matrix with increasing number of particles.

Better estimates of  $N_n$  and  $\Delta_c$  should take into account the fact that, close to the Anderson transition, the structure of the one-particle eigenstates reduces the number of effectively coupled states, while enhancing their correlations and hence the mean transition matrix elements [16]. The latter in their turn, are directly related to the spacing between directly coupled states by the relation obeyed at the transition  $Q_c \sim \Delta_c$ .

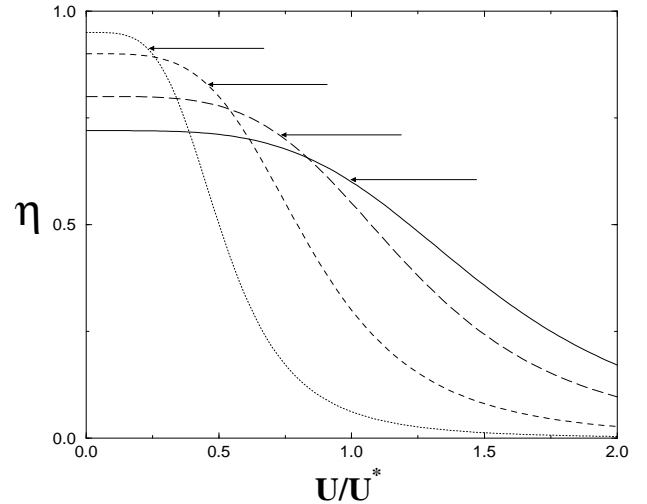


FIG. 3. Schematic evolution of  $\eta$  as a function of the renormalized interaction  $U/U^*$  in the regime  $n_{eff} \approx 2\sqrt{\delta\epsilon/\Delta_1} < n$  ( $U^*$  corresponds to the smaller system size.). The effective number of interacting particles increases with the system size, and in the process,  $\eta_c(n_{eff})$  increases as indicated by the arrows and the curves no longer intersect at one point. In the other regime studied in this article,  $\delta\epsilon > \Delta_1 n$ , the number of effectively interacting particle is invariant upon increase of the system size and the transition is unambiguously determined by the intersection of all  $\eta$ -curves (see Fig.1).

Our interpretation relates the level repulsion at the transition to the  $n$ -body correlation volume  $B_1/\Delta_c$ , i.e. the volume inside which  $n$ -body states have a strong overlap and are therefore directly connected by the interaction. Indeed in the three-dimensional one particle Anderson model, this volume goes like  $L^{d-\omega}$  with a multifractal exponent  $\omega \approx 1.3$  [17]. Hence we have  $N_1 \sim L^\omega$  and therefore the critical statistics is intermediate between Poisson ( $N_1 \approx L^d$ ) and Wigner-Dyson ( $N_1 \approx 1$ ). Moreover in the case of two particles, the two-body interaction connects all states together and  $N_2$  can be reduced only by the one-particle correlations. Consequently, the critical statistics is very well fitted by the semi-Poisson  $P_{SP}(s) = 4s \exp(-2s)$ , as can be the case for the one-particle model [15]. As  $n$  increases, the Hamiltonian ma-

trix becomes sparser and sparser and the critical spacings distribution exhibits less and less level repulsion.

In the thermodynamic (TD) limit, for such Fermi systems, it is natural to define a temperature as  $n_{eff}T = T^2/\Delta_1 \sim \delta\epsilon$ , and as  $T$  must be constant as  $L \rightarrow \infty$ , this implies that  $T/B_1 \approx n_{eff}\Delta_1/B_1 = \text{const}$ . Two different regimes are then of interest:

- The one-particle critical regime close to the Anderson transition, where the system size is significantly smaller than the one-particle localization length. There, both the one-particle correlation volume and  $n_{eff}$  increase as  $\sim L^{d-\omega}$  until  $L \approx l_1$  and since  $l_1$  can be very large, so is  $n_{eff}$  and there is almost no level repulsion at the transition in the TD limit. In this case the scaling at the transition is not observable since  $\eta_c$  increases with  $n$  and the  $\eta$ -curves intersect at different points (see Fig.3).

- The one-particle localized regime  $1 \ll l_1 < L$ , where we have  $\Delta_1 \approx B_1/l_1^d = \text{const}$ . This regime corresponds to the results presented in Fig.1 for which  $n_{eff}$  is constant and one has a fixed-point at a well-defined value of  $U_c$ .

Since it is expected that a large interaction will ultimately transform the system back into an insulator, it is important to estimate the parametrical width of the metallic phase. In this case, it seems natural to carry out the substitution  $\Delta_1 \rightarrow Q$  in (2) so that in the localized regime with  $n_{eff} = \text{const}$ , the condition  $Q_c \sim \Delta_c$  leads to a metallic phase in the range

$$\frac{\Delta_1}{n_{eff}^2 T} \leq \frac{Q}{\Delta_1} \leq \frac{n_{eff}^2 T}{\Delta_1} \quad (4)$$

Since  $n_{eff}^2 T = \delta\epsilon \gg \Delta_1$ , the regime defined by (4) is parametrically large enough to allow for a metallic phase. The backtransition at high  $Q$  occurs before the Wigner crystallization, i.e. it is induced by the lowering of the density of states much in the same way as the transition described above, and occurs before the opening of a gap.

Without the constraint of a filled Fermi sea ( $P_b = 1$ ), levels are filled from the bottom of the Anderson band where the decrease of  $l_1$  implies an increase of  $\Delta_1$ . Correspondingly, the metallic regime determined by (4) is reduced unless the disorder is lowered below  $W_c$  so as to get the one-particle mobility edge close to the Fermi level, and increase the localization length of the one-particle band-tail states. In this case, the situation should be similar to the one reported above, i.e. the one-particle correlation volume is sufficiently large to allow the mixing of a large number of states and a metal-insulator transition should occur.

In conclusion we successfully applied the method developed in [5] to disordered many-body systems. We showed the existence of a continuous metal-insulator phase transition where the critical level spacings distribution is different from the one at the Anderson transition. These considerations quite naturally lead to the identification of the metallic phase with the appearance of quantum

chaos and the mixing of directly coupled states.

It is a pleasure to thank A. D. Stone for many suggestions that helped improve this paper. Very interesting discussions with M. Janssen are also acknowledged. Numerical computations were performed at the Swiss Center for Scientific Computing. Work supported by the Swiss National Science Foundation.

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